Determinants

1.

$$\begin{vmatrix} x+1 & x-1 \\ x^2+x+1 & x^2-x+1 \end{vmatrix} \text{ is equal to :}$$

(2024)

(A) $2x^3$

(B) 2

(C) 0

(D) $2x^3 - 2$

Ans. (B) 2

Previous Years' CBSE Board Questions

4.2 Determinant

MCQ

1. The value of the determinant $\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$ is
(a) 47 (b) −79 (c) 49 (d) −51
(2023)

2. If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is
(a) 1 (b) 2 (c) 3 (d) 4
(2023)

3. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 27$, then the value of α is
(a) ± 1 (b) ± 2 (c) $\pm\sqrt{5}$ (d) $\pm\sqrt{7}$
(Term I, 2021-22) U

4. If $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & -3 \\ 9 & 6 & -2 \end{vmatrix} = 0$, then the value of x is
(a) 3 (b) 5 (c) 7 (d) 9
(Term I, 2021-22) R

5. The determinant $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$ is equal to
(a) $k(3y + k^2)$ (b) $3y + k^3$
(c) $3y + k^2$ (d) $k^2(3y + k)$
(Term I, 2021-22)

6. The value of $\begin{vmatrix} \lfloor 1 & \lfloor 2 & \lfloor 3 \\ 2\lfloor 2 & 3\lfloor 3 & 4\lfloor 4 \\ \lfloor 3 & \lfloor 4 & \lfloor 5 \end{vmatrix}$ is
(a) 12 (b) −12 (c) 24 (d) −24
(Term I, 2021-22)

7. If A is a non-singular square matrix of order 3 such that $A^2 = 3A$, then value of $|A|$ is
(a) − 3 (b) 3
(b) 9 (d) 27 *(2020)*

8. The roots of the equation $\begin{vmatrix} x & 0 & 8 \\ 4 & 1 & 3 \\ 2 & 0 & x \end{vmatrix} = 0$ are
(a) − 4, 4 (b) 2, − 4 (c) 2, 4 (d) 2, 8
(2020 C)

9. If A is a square matrix of order 3 and $|A| = 5$, then the value of $|2A'|$ is
(a) −10 (b) 10
(c) −40 (d) 40 *(2020)* U

10. If A is a skew-symmetric matrix of order 3, then the value of $|A|$ is
(a) 3 (b) 0
(c) 9 (d) 27 *(2020)*

11. If A is a 3×3 matrix such that $|A| = 8$, then $|3A|$ equals
(a) 8 (b) 24
(c) 72 (d) 216 *(2020)* U

VSA (1 mark)

12. If $A = \begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix}$, then $|AB|$ = ______.
(2020 C)

13. If A and B are square matrices each of order 3 and $|A| = 5$, $|B| = 3$, then the value of $|3AB|$ is ______. *(2020)*

14. If A and B are square matrices of the same order 3, such that $|A| = 2$ and $AB = 2I$, write the value of $|B|$.
(Delhi 2019) An

15. Find the maximum value of
$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$. *(Delhi 2016)*

16. If $x \in N$ and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of x.
(AI 2016)

17. If $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8$, write the value of x.
(Foreign 2016) An

18. Write the value of $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$. *(AI 2015)*

19. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}$, write the value of $|AB|$.
(Delhi 2015C)

20. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, write the value of x.
(Delhi 2014) U

21. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, find the value of x. *(AI 2014)*

22. If A is a 3×3 matrix, $|A| \neq 0$ and $|3A| = k|A|$, then write the value of k. *(Foreign 2014)*

23. Write the value of the determinant $\begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix}$.
(Delhi 2014C)

24. Write the value of $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$. *(AI 2014C)* Ap

SA II (3 marks)

25. Show that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ. *(2023)*

4.4 Minors and Cofactors

VSA (1 mark)

26. Find the cofactors of all the elements of $\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$. *(2020)* Ap

27. Find the cofactor of the element a_{23} of the determinant $\begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$. *(2019C)*

28. If $A = \begin{bmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$, then write the cofactor of the element a_{21} of its 2nd row. *(Foreign 2015)*

4.5 Adjoint and Inverse of a Matrix

MCQ

29. The inverse of $\begin{bmatrix} -4 & 3 \\ 7 & -5 \end{bmatrix}$ is

(a) $\begin{bmatrix} -5 & 3 \\ 7 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 3 \\ 7 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} -5 & 7 \\ 3 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -3 \\ -7 & -4 \end{bmatrix}$

(Term I, 2021-22) Ap

30. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 59 & 69 & -1 \end{bmatrix}$, then A^{-1}

(a) is A (b) is $(-A)$

(c) is A^2 (d) does not exist

(Term I, 2021-22)

31. If $A = \begin{bmatrix} 1 & -2 & 4 \\ 2 & -1 & 3 \\ 4 & 2 & 0 \end{bmatrix}$ is the adjoint of a square matrix B, then B^{-1} is equal to

(a) $\pm A$ (b) $\pm\sqrt{2}A$ (c) $\pm\frac{1}{\sqrt{2}}B$ (d) $\pm\frac{1}{\sqrt{2}}A$

(Term I, 2021-22) Cr

32. If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then

(a) $a = 1 = b$ (b) $a = \cos 2\theta, b = \sin 2\theta$

(c) $a = \sin 2\theta, b = \cos\theta$ (d) $a = \cos\theta, b = \sin\theta$

(Term I, 2021-22)

33. If $A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then the value of $|\text{adj } A|$ is

(a) 64 (b) 16 (c) 0 (d) −8

(2020)

34. If A is a square matrix of order 3, such that $A(\text{adj } A) = 10I$, then $|\text{adj } A|$ is equal to

(a) 1 (b) 10 (c) 100 (d) 101

(2020)

VSA (1 mark)

35. If A is a square matrix of order 3 such that $A(\text{adj } A) = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then find $|A|$. *(2021 C)*

36. If $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 3 \\ 3 & 3 & 5 \end{bmatrix}$, then find $A(\text{adj } A)$. *(2020)* Ap

37. If A is a square matrix of order 3 with $|A| = 9$, then write the value of $|2 \cdot \text{adj } A|$. *(AI 2019)*

38. If A is a 3 × 3 invertible matrix, then what will be the value of k if $\det(A^{-1}) = (\det A)^k$? *(Delhi 2017)* R

39. If for any 2 × 2 square matrix A, $A(\text{adj } A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of $|A|$. *(AI 2017)*

40. In the interval $\pi/2 < x < \pi$, find the value of x for which the matrix $\begin{bmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{bmatrix}$ is singular. *(AI 2015C)*

41. Find $(\text{adj } A)$, if $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$. *(Delhi 2014C)* Ev

SA I (2 marks)

42. For the matrix $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$, verify the following:
$A(\text{adj } A) = (\text{adj } A)A = |A|I$ *(2020C)*

43. Find $(AB)^{-1}$ if $A = \begin{bmatrix} 1 & 0 \\ -4 & 2 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$. *(2020)* Ap

44. Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute I^{-1} and show that $2A^{-1} = 9I - A$. *(2018)* Cr

LA I (4 marks)

45. Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹160. From the same shop Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹250.

(i) Convert the given above situation into a matrix equation of the form $AX = B$.
(ii) Find $|A|$.
(iii) Find A^{-1}.

OR

Determine $P = A^2 - 5A$. (2023)

46. If $A^{-1}=\begin{bmatrix}3 & -1 & 1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{bmatrix}$ and $B=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$, find $(AB)^{-1}$. (2021C)

47. If $A=\begin{bmatrix}1 & -2 & 3\\ 0 & -1 & 4\\ -2 & 2 & 1\end{bmatrix}$, find $(A')^{-1}$. (Delhi 2015)

48. Find the adjoint of the matrix $A = \begin{bmatrix}-1 & -2 & -2\\ 2 & 1 & -2\\ 2 & -2 & 1\end{bmatrix}$ and hence show that $A\cdot(\text{adj } A) = |A|I_3$. (AI 2015) Cr

49. If $A=\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}$ and I is the identity matrix of order 2, then show that $A^2 = 4A - 3I$. Hence find A^{-1}. (Foreign 2015)

50. Find the inverse of the matrix $A = \begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}$. (Delhi 2015C) Ap

51. If $A=\begin{bmatrix}2 & 3\\ 1 & -4\end{bmatrix}$ and $B=\begin{bmatrix}1 & -2\\ -1 & 3\end{bmatrix}$, then verify that $(AB)^{-1} = B^{-1}A^{-1}$. (AI 2015C)

LA II (5/6 marks)

52. If $A=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$, find adj A and verify that $A(\text{adj } A) = (\text{adj } A)A = |A|\, I_3$. (Foreign 2016) Cr

4.6 Applications of Determinants and Matrices

LA I (4 marks)

53. If $\begin{bmatrix}3 & 4 & 2\\ 0 & 2 & -3\\ 1 & -2 & 6\end{bmatrix}$, find A^{-1}.
Hence, solve the following system of equations :
$3x + 4y + 2z = 8$
$2y - 3z = 3$
$x - 2y + 6z = -2$ (2021C)

54. The monthly incomes of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5 : 7. If each saves ₹ 15,000 per month, find their monthly incomes using matrix method. This problem reflects which value? (Delhi 2016) Cr

55. A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received ₹ 2,800 as interest. However, if trust had interchanged money in bonds they would have got ₹ 100 less as interest. Using matrix method, find the amount invested by the trust. Interest received on this amount will be given to Helpage India as donation. Which value is reflected in this question? (AI 2016)

56. A coaching institute of English (Subject) conducts classes in two batches I and II and fees for rich and poor children are different. In batch I, it has 20 poor and 5 rich children and total monthly collection is ₹ 9,000, whereas in batch II, it has 5 poor and 25 rich children and total monthly collection is ₹ 26,000. Using matrix method, find monthly fees paid by each child of two types. What values the coaching institute is inculcating in the society? (Foreign 2016) Cr

57. Two schools A and B decided to award prizes to their students for three values, team spirit, truthfulness and tolerance at the rate of ₹ x, ₹ y and ₹ z per student respectively. School A, decided to award a total of ₹ 1,100 for the three values to 3, 1 and 2 students respectively while school B decided to award ₹ 1,400 for the three values to 1, 2 and 3 students respectively. If one prize for all the three values together amount to ₹ 600 then
(i) Represent the above situation by a matrix equation after forming linear equations.
(ii) Is it possible to solve the system of equations so obtained using matrices?
(iii) Which value you prefer to be rewarded most and why? (Delhi 2015C) Cr

LA II (5/6 marks)

58. Find the inverse of the matrix $A=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$.
Using the inverse, A^{-1}, solve the system of linear equations
$x - y + 2z = 1;\ 2y - 3z = 1;\ 3x - 2y + 4z = 3.$ (2023)

59. If $A = \begin{bmatrix}1 & 2 & -3\\ 3 & 2 & -2\\ 2 & -1 & 1\end{bmatrix}$, then find A^{-1} and use it to solve the following system of the equations :
$x + 2y - 3z = 6,\ 3x + 2y - 2z = 3,$
$2x - y + z = 2$ (2020)

60. Solve the following system of equations by matrix method :
$x - y + 2z = 7,$
$2x - y + 3z = 12,$
$3x + 2y - z = 5$ (2020)

61. If $A=\begin{bmatrix}2 & -3 & 5\\3 & 2 & -4\\1 & 1 & -2\end{bmatrix}$, then find A^{-1}.

Using A^{-1}, solve the following system of equations :
$2x - 3y + 5z = 11, 3x + 2y - 4z = -5$
$x + y - 2z = -3$ *(NCERT, 2020, 2018)* Ev

62. If $\begin{bmatrix}0 & 1 & 2\\1 & 2 & 3\\3 & 1 & 1\end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the system of equations
$y + 2z = 5$
$x + 2y + 3z = 10$
$3x + y + z = 9$ *(2019C)*

63. If $A=\begin{bmatrix}1 & 1 & 1\\1 & 0 & 2\\3 & 1 & 1\end{bmatrix}$, find A^{-1}. Hence, solve the system of equations $x + y + z = 6, x + 2z = 7, 3x + y + z = 12$. *(Delhi 2019)*

64. Using matrices, solve the following system of linear equations:
$x + 2y - 3z = -4, 2x + 3y + 2z = 2$
$3x - 3y - 4z = 11$ *(AI 2019)*

65. Use product $\begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{bmatrix}$ to solve the system of equations $x + 3z = 9, -x + 2y - 2z = 4,$
$2x - 3y + 4z = -3$ *(Delhi 2017)*

66. Determine the product

$\begin{bmatrix}-4 & 4 & 4\\-7 & 1 & 3\\5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\1 & -2 & -2\\2 & 1 & 3\end{bmatrix}$ and use it to solve the system of equations $x - y + z = 4,$
$x - 2y - 2z = 9, 2x + y + 3z = 1.$ *(AI 2017)* Ev

67. A shopkeeper has 3 varieties of pens 'A', 'B' and 'C'. Meenu purchased 1 pen of each variety for a total of ₹ 21. Jeevan purchased 4 pens of 'A' variety, 3 pens of 'B' variety and 2 pens of 'C' variety for ₹ 60. While Shikha purchased 6 pens of 'A' variety, 2 pens of 'B' variety and 3 pens of 'C' variety for ₹ 70. Using matrix method, find cost of each variety of pen. *(AI 2016)* Cr

68. Two schools P and Q want to award their selected students on the values of discipline, politeness and punctuality. The school P wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to its 3, 2 and 1 students with a total award money of ₹ 1,000. School Q wants to spend ₹ 1,500 to award its 4, 1 and 3 students on the respective values (by giving the same award money for the three values as before). If the total amount of awards for one prize on each value is ₹ 600, using matrices, find the award money for each value.
Apart from the above three values, suggest one more value for awards. *(Delhi 2014)* Cr

69. Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school A wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to 3, 2 and 1 students respectively with a total award money of ₹ 1,600. School B wants to spend ₹ 2,300 to award its, 4, 1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount of award for one prize on each value is ₹ 900, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for award. *(AI 2014)* Cr

70. Two schools P and Q want to award their selected students on the values of tolerance, kindness and leadership. The school P wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to its 3, 2 and 1 students respectively with a total award money of ₹ 2200. School Q wants to spend ₹ 3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money for the three values as school P). If the total amount of award for one prize on each value is ₹ 1200, using matrices, find the award money for each value.
Apart from the above these three values, suggest one more value which should be considered for award. *(Foreign 2014)*

71. A total amount of ₹ 7,000 is deposited in three different savings bank accounts with annual interest rates of 5%, 8% and $8\frac{1}{2}$% respectively. The total annual interest from these three accounts is ₹ 550. Equal amounts have been deposited in the 5% and 8% savings accounts. Find the amount deposited in each of the three accounts, with the help of matrices. *(Delhi 2014C)* Cr

72. Two schools, P and Q, want to award their selected students for the values of sincerity, truthfulness and hard work at the rate of ₹ x, ₹ y and ₹ z for each respective value per student. School P awards its 2, 3 and 4 students on the above respective values with a total prize money of ₹ 4,600. School Q wants to award its 3, 2 and 3 students on the respective values with a total award money of ₹ 4,100. If the total amount of award money for one prize on each value is ₹ 1,500, using matrices find the award money for each value. Suggest one other value which the school can consider for awarding the students. *(AI 2014C)*

CBSE Sample Questions

4.2 Determinant

MCQ

1. If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, then the possible value(s) of 'x' is/are
   (a) 3 (b) $\sqrt{3}$
   (c) $-\sqrt{3}$ (d) $\sqrt{3}, -\sqrt{3}$ *(2022-23)*

2. If A is a square matrix of order 3, $|A'| = -3$, then $|AA'| =$
   (a) 9 (b) −9 (c) 3 (d) −3 *(2022-23)*

3. Value of k, for which $A = \begin{bmatrix} k & 8 \\ 4 & 2k \end{bmatrix}$ is a singular matrix is
   (a) 4 (b) −4 (c) ±4 (d) 0
   (Term I, 2021-22) Cr

4. Given that A is a non-singular matrix of order 3 such that $A^2 = 2A$, then value of $|2A|$ is
   (a) 4 (b) 8 (c) 64 (d) 16
   (Term I, 2021-22)

SA I (2 marks)

5. If A is a square matrix of order 3 such that $A^2 = 2A$, then find the value of $|A|$. *(2020-21)* Cr

4.4 Minors and Cofactors

MCQ

6. Given that $A = [a_{ij}]$ is a square matrix of order 3×3 and $|A| = -7$, then the value of $\sum_{i=1}^{3} a_{i2}A_{i2}$, where A_{ij} denotes the cofactor of element a_{ij} is
   (a) 7 (b) −7
   (c) 0 (d) 49 *(Term I, 2021-22)*

VSA (1 mark)

7. Let $A = [a_{ij}]$ be a square matrix of order 3×3 and $|A| = -7$. Find the value of $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23}$ where A_{ij} is the cofactor of element a_{ij}.
   (2020-21) Cr

4.5 Adjoint and Inverse of a Matrix

MCQ

8. If A is a square matrix of order 3 and $|A| = 5$, then $|\text{adj } A| =$
   (a) 5 (b) 25 (c) 125 (d) $\frac{1}{5}$ *(2022-23)*

9. Given that A is a square matrix of order 3 and $|A| = -4$, then $|\text{adj } A|$ is equal to
   (a) −4 (b) 4
   (c) −16 (d) 16
   (Term I, 2021-22)

10. For matrix $A = \begin{bmatrix} 2 & 5 \\ -11 & 7 \end{bmatrix}$, $(\text{adj } A)'$ is equal to
   (a) $\begin{bmatrix} -2 & -5 \\ 11 & -7 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & 5 \\ 11 & 2 \end{bmatrix}$
   (c) $\begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix}$
   (Term I, 2021-22) Ap

11. For $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then $14A^{-1}$ is given by
   (a) $14\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$
   (c) $2\begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix}$ (d) $2\begin{bmatrix} -3 & -1 \\ 1 & -2 \end{bmatrix}$
   (Term I, 2021-22) Ap

VSA (1 mark)

12. Given that A is a square matrix of order 3×3 and $|A| = -4$. Find $|\text{adj } A|$. *(2020-21)*

4.6 Applications of Determinants and Matrices

LA II (5/6 marks)

13. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}. Use A^{-1} to solve the following system of equations. *(2022-23)*
   $2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$

14. If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1}. Hence solve the system of equations;
   $x - 2y = 10$, $2x - y - z = 8$, $-2y + z = 7$ *(2020-21)* Ev

15. Evaluate the product AB, where
   $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$
   Hence solve the system of linear equations
   $x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 7$ *(2020-21)* Cr

Detailed SOLUTIONS

Previous Years' CBSE Board Questions

1. (a): Let $|A| = \begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$

Expanding along R_1, we get

$|A| = 2(1-8) - 7(1-10) + 1(8-10)$

$= 2(-7) - 7(-9) + 1(-2)$

$= -14 + 63 - 2 = 47$

2. (d): $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0 \Rightarrow \alpha(2-4) - 3(1-1) + 4(4-2) = 0$

$\Rightarrow -2\alpha + 8 = 0 \Rightarrow 2\alpha = 8 \Rightarrow \alpha = 4$

3. (d): Given, $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ ...(i)

$|A^3| = 27$

$\Rightarrow |A|^3 = 27$ [$\because |A^n| = |A|^n$] $\Rightarrow |A| = 3$ ...(ii)

From (i) and (ii), we get

$\Rightarrow \alpha^2 - 4 = 3 \Rightarrow \alpha^2 = 7 \Rightarrow \alpha = \pm\sqrt{7}$

4. (d): We have, $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & -3 \\ 9 & 6 & -2 \end{vmatrix} = 0$

$\Rightarrow 5(-2x + 18) - 3(14 + 27) - 1(-42 - 9x) = 0$

$\Rightarrow -x + 9 = 0$

$\Rightarrow x = 9$

Concept Applied

➲ $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$

5. (d): Let $D = \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$

$= (y+k)((y+k)^2 - y^2) - y(y^2 + ky - y^2) + y(y^2 - y^2 - yk)$

$= k^3 + 3k^2y = k^2(k + 3y)$

6. (d): Given, $\begin{vmatrix} 1! & 2! & 3! \\ 2\,2! & 3\,3! & 4\,4! \\ 3! & 4! & 5! \end{vmatrix} = \begin{vmatrix} 1 & 2 & 6 \\ 4 & 18 & 96 \\ 6 & 24 & 120 \end{vmatrix}$

$= 1(2160 - 2304) - 2(480 - 576) + 6(96 - 108)$

$= -144 - 2(-96) + 6(-12) = -144 + 192 - 72 = -24$

7. (d): Given $A^2 = 3A$

$\Rightarrow |A^2| = |3A| \Rightarrow |A|^2 = 3^3|A| \Rightarrow |A|^2 - 27|A| = 0$

$\Rightarrow |A|\,[|A| - 27] = 0$

As, A is a non-singular matrix

$\therefore\ |A| \neq 0 \Rightarrow |A| - 27 = 0 \Rightarrow |A| = 27$

8. (a): Given, $\begin{vmatrix} x & 0 & 8 \\ 4 & 1 & 3 \\ 2 & 0 & x \end{vmatrix} = 0$

Expanding along R_1, we get

$x(x-0) - 0 + 8(0-2) = 0$

$\Rightarrow x^2 - 16 = 0 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

9. (d): Given, A is a 3×3 matrix and $|A| = 5$

Now, $|2A'| = 2^3|A'| = 2^3|A| = 8 \times 5 = 40$

Answer Tips

➲ In a square matrix, $|A'| = |A|$

10. (b) : We have, $A^T = -A$ [$\because$ A is skew-symmetric matrix]

$\therefore\ |A^T| = |-A| \Rightarrow |A| = (-1)^3|A|$ [$\because$ A is of order 3]

$\Rightarrow |A| = -|A| \Rightarrow 2|A| = 0 \Rightarrow |A| = 0$

11. (d) : We have, $|3A| = 3^3|A| = 3^3 \cdot 8$ [Given $|A| = 8$]

$= 27 \cdot 8 = 216$

12. If $A = \begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix}$, then

$AB = \begin{bmatrix} 3 & 101 \\ 2 & 34 \end{bmatrix}$

Now, $|AB| = \begin{vmatrix} 3 & 101 \\ 2 & 34 \end{vmatrix} = 102 - 202 = -100$

13. We have, $|A| = 5$, $|B| = 3$

Now, $|3AB| = 3^3|AB|$ ($\because$ Order of AB is 3)

$= 3^3|A||B| = 3^3 \times 5 \times 3 = 405$

14. We have, $AB = 2I \Rightarrow |AB| = |2I| \Rightarrow |A|\,|B| = 2^3|I|$

[$\because$ A and B are order 3]

$\Rightarrow 2|B| = 8$ [$\because |A| = 2$]

$\Rightarrow |B| = 4$

15. Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$

$\Rightarrow \Delta = 1[(1+\sin\theta)(1+\cos\theta) - 1] - 1(1 + \cos\theta - 1) + 1(1 - 1 - \sin\theta)$

$= 1 + \cos\theta + \sin\theta + \sin\theta\cos\theta - 1 - \cos\theta - \sin\theta = \sin\theta\cos\theta$

$\therefore$ Maximum value of Δ is $\frac{1}{2}$.

16. Given, $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$

$\Rightarrow (x+3)(2x) - (-2)(-3x) = 8$

$\Rightarrow 2x^2 + 6x - 6x = 8 \Rightarrow 2x^2 = 8$

$\Rightarrow x^2 = 4 \Rightarrow x = 2$ [$x \neq -2,\ \because x \in N$]

17. Given, $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8$

$\Rightarrow x(-x^2 - 1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta) = 8$

$\Rightarrow \quad -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \sin\theta\cos\theta + x\cos^2\theta = 8$

$\Rightarrow \quad -x^3 - x + x(\sin^2\theta + \cos^2\theta) = 8$

$\Rightarrow \quad -x^3 - x + x = 8 \Rightarrow x^3 + 8 = 0$

$\Rightarrow \quad (x+2)(x^2 - 2x + 4) = 0 \Rightarrow x + 2 = 0$

$[\because x^2 - 2x + 4 > 0 \ \forall x]$

$\Rightarrow \quad x = -2$

18. $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$

$= (x+y)(-3x+3y) - (y+z)(-3z+3y) + (z+x)(-3z+3x)$

$= 3[(y+x)(y-x) - (y+z)(y-z) + (x+z)(x-z)]$

$= 3[y^2 - x^2 - y^2 + z^2 + x^2 - z^2] = 0$

19. Given that $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}$

$\therefore \quad AB = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 4 & 8 \end{bmatrix}$

$\therefore \quad |AB| = \begin{vmatrix} -1 & 5 \\ 4 & 8 \end{vmatrix} = (-1)\cdot 8 - 4\cdot 5 = -28.$

20. Given, $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$

$\Rightarrow \quad 2x^2 - 40 = 18 + 14$

$\Rightarrow \quad 2x^2 = 72 \Rightarrow x^2 = 36 \Rightarrow x = \pm 6$

21. Given, $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$

$\Rightarrow \quad 12x + 14 = 32 - 42$

$\Rightarrow \quad 12x = -10 - 14 = -24 \Rightarrow x = -2.$

22. We have, $|3A| = k|A|$

A is 3×3 matrix, so we have

$3^3|A| = k|A|$ [Using $|mA| = m^n|A|$, where n is order of A]

$\Rightarrow \quad k = 27.$

23. $\begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix} = p^2 - (p-1)(p+1) = p^2 - (p^2 - 1) = 1.$

24. Let $\Delta = \begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$

$\Rightarrow \quad \Delta = 2[8(86) - 9(75)] - 7[3(86) - 5(75)] + 65[3(9) - 5(8)]$

$= 2(688 - 675) - 7(258 - 375) + 65(27 - 40)$

$= 2(13) - 7(-117) + 65(-13) = 26 + 819 - 845 = 0$

25. We have, $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$

$= x(-x^2 - 1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$

$= -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \sin\theta\cos\theta + x\cos^2\theta$

$= -x^3 - x + x(\sin^2\theta + \cos^2\theta) = -x^3 - x + x$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= -x^3$, which is independent of θ.

26. Let $A = \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$

Cofactor of $1 = 3$, Cofactor of $-2 = -4$.

Cofactor of $4 = 2$, Cofactor of $3 = 1$.

Concept Applied

- Cofactor, $A_{ij} = (-1)^{i+j} M_{ij}$, where M_{ij} is minor of a_{ij}.

27. Given, $\begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

Cofactor of $a_{23} = (-1)^{2+3}\begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = (-1)(10-3) = -7$

28. We have, $A = \begin{bmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$

$\therefore \quad$ Cofactor of $a_{21} = (-1)^{2+1}\begin{vmatrix} 6 & -3 \\ -7 & 3 \end{vmatrix} = -1(18 - 21) = 3$

29. **(b)**: Given, $A = \begin{bmatrix} -4 & 3 \\ 7 & -5 \end{bmatrix}$

$\therefore \quad |A| = 20 - 21 = -1$

And adj $A = \begin{bmatrix} -5 & -7 \\ -3 & -4 \end{bmatrix}^T = \begin{bmatrix} -5 & -3 \\ -7 & -4 \end{bmatrix}$

$\therefore \quad A^{-1} = \frac{1}{|A|}(\text{adj}A) = \begin{bmatrix} 5 & 3 \\ 7 & 4 \end{bmatrix}$

30. **(a)**: Given, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 59 & 69 & -1 \end{bmatrix}$

Here, $|A| = -1$

And adj$A = \begin{bmatrix} -1 & 0 & -59 \\ 0 & -1 & -69 \\ 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ -59 & -69 & 1 \end{bmatrix}$

$\therefore \ A^{-1} = \frac{1}{|A|}(\text{adj}A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 59 & 69 & -1 \end{bmatrix} = A$

Key Points

- The adjoint of a square matrix is the transpose of the matrix of cofactors.

31. **(d)**: $|A| = \begin{vmatrix} 1 & -2 & 4 \\ 2 & -1 & 3 \\ 4 & 2 & 0 \end{vmatrix}$

$= 1(0-6) + 2(0-12) + 4(4+4) = 2 \quad$ ...(i)

Given, $A = \text{adj}B$

$\Rightarrow \quad |A| = |\text{adj}B| \Rightarrow |\text{adj}B| = 2 \quad$ (Using (i))

$\Rightarrow \quad |B|^2 = 2 \quad [\because |\text{adj}B| = |B|^{3-1}$, where B is 3×3 matrix$]$

$\Rightarrow \quad |B| = \pm\sqrt{2}$

$\therefore \ B^{-1} = \pm\frac{1}{\sqrt{2}}A \left[\because B^{-1} = \frac{1}{|B|}(\text{adj}B)\right]$

Concept Applied

- $|\text{adj } A| = |A|^{n-1}$, where n is the order of matrix A.

32. (b): We have,

$$\begin{bmatrix}1 & -\tan\theta\\ \tan\theta & 1\end{bmatrix}\begin{bmatrix}1 & \tan\theta\\ -\tan\theta & 1\end{bmatrix}^{-1}=\begin{bmatrix}a & -b\\ b & a\end{bmatrix}$$

Now, $\begin{bmatrix}1 & \tan\theta\\ -\tan\theta & 1\end{bmatrix}^{-1}=\dfrac{1}{\sec^2\theta}\begin{bmatrix}1 & -\tan\theta\\ \tan\theta & 1\end{bmatrix}$

$$=\begin{bmatrix}\cos^2\theta & -\tan\theta\cos^2\theta\\ \cos^2\theta\tan\theta & \cos^2\theta\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1 & -\tan\theta\\ \tan\theta & 1\end{bmatrix}\begin{bmatrix}\cos^2\theta & -\tan\theta\cos^2\theta\\ \cos^2\theta\tan\theta & \cos^2\theta\end{bmatrix}=\begin{bmatrix}a & -b\\ b & a\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}\cos^2\theta-\cos^2\theta\tan^2\theta & -2\tan\theta\cos^2\theta\\ 2\tan\theta\cos^2\theta & \cos^2\theta-\cos^2\theta\tan^2\theta\end{bmatrix}=\begin{bmatrix}a & -b\\ b & a\end{bmatrix}$$

$\therefore$ $a=\cos^2\theta-\cos^2\theta\tan^2\theta$ and $b=2\tan\theta\cos^2\theta$

$\Rightarrow a=\cos^2\theta\left(1-\dfrac{\sin^2\theta}{\cos^2\theta}\right)$ and $b=\dfrac{2\sin\theta}{\cos\theta}\cdot\cos^2\theta$

$\Rightarrow a=\cos^2\theta-\sin^2\theta=\cos 2\theta$ and $b=2\sin\theta\cos\theta=\sin 2\theta$

33. (a) : We have, $|A|=\begin{vmatrix}-2 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & -2\end{vmatrix}$

$|A|=-2(4-0)-0+0=-8$

$\therefore$ $|\text{adj } A|=(-8)^2=64$

Answer Tips

- We know that $|A \text{ adj } A|=|A|^n$, where n is the order of A.

34. (c): Given, $A(\text{adj } A)=10\,I$

$\Rightarrow |A(\text{adj}) A)|=|10\,I|$

$\Rightarrow |A|\,|\text{adj } A|=10^3\,|I|$ ...(i)

$\Rightarrow |A||A|^2=10^3$

[$\because$ A is matrix of order n, then $|\text{adj } A|=|A|^{n-1}$ and $n=3$]

$\Rightarrow |A|^3=10^3$

$\Rightarrow |A|=10$

Now, from (i), we get

$|(\text{adj}) A)|=|A|^2=10^2=100$

35. Given $A(\text{adj } A)=\begin{vmatrix}-2 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & -2\end{vmatrix}$

$$|A(\text{adj } A)|=\begin{vmatrix}-2 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & -2\end{vmatrix}=(-2)^3=-8$$

$\Rightarrow |A|^3=-8$ [$\because |A \text{ adj } A|=|A|^n$]

$\Rightarrow |A|=-2$

36. Given, $A=\begin{bmatrix}2 & 0 & 0\\ -1 & 2 & 3\\ 3 & 3 & 5\end{bmatrix}$

Now, $A(\text{adj } A)=|A|=\begin{vmatrix}2 & 0 & 0\\ -1 & 2 & 3\\ 3 & 3 & 5\end{vmatrix}$

$=2(10-9)-0+0=2(1)=2$

37. Given, $|A|=9$

$\therefore$ $|2\cdot\text{adj } A|=2^3|A|^2=2^3(9)^2=8\times 81=648$

Concept Applied

- We know that, $|k \text{ adj } A|=k^n|A|^{n-1}$, where n is the order of the matrix A.

38. Given that, $\det(A^{-1})=(\det A)^k$

i.e., $|A^{-1}|=|A|^k$

We know that $|A^{-1}|=\dfrac{1}{|A|}=|A|^{-1}$

$\therefore$ $k=-1$

39. We have, $A(\text{adj } A)=\begin{bmatrix}8 & 0\\ 0 & 8\end{bmatrix}$

$|A|\cdot I=8\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ [$\because A(\text{adj } A)=|A|\times I$]

$\Rightarrow |A|=8$

40. For the matrix $\begin{bmatrix}2\sin x & 3\\ 1 & 2\sin x\end{bmatrix}$ to be singular, its determinant = 0

$\therefore$ $\begin{vmatrix}2\sin x & 3\\ 1 & 2\sin x\end{vmatrix}=0$

$\Rightarrow 4\sin^2 x-3=0 \Rightarrow \sin^2 x=\dfrac{3}{4} \Rightarrow \sin x=\pm\dfrac{\sqrt{3}}{2}$

$\therefore$ $x=\dfrac{2\pi}{3}\left(\because \dfrac{\pi}{2}<x<\pi\right)$

41. Here, $A=\begin{bmatrix}5 & 2\\ 7 & 3\end{bmatrix}$

Cofactor of matrix $A=\begin{bmatrix}3 & -7\\ -2 & 5\end{bmatrix}$

$\therefore$ $\text{adj } A=\begin{bmatrix}3 & -7\\ -2 & 5\end{bmatrix}^T=\begin{bmatrix}3 & -2\\ -7 & 5\end{bmatrix}$

42. Given, $A=\begin{bmatrix}2 & 3\\ -4 & -6\end{bmatrix} \Rightarrow |A|=-12+12=0$

$\therefore$ A is a singular matrix

$\text{adj } A=\begin{bmatrix}-6 & 4\\ -3 & 2\end{bmatrix}'=\begin{bmatrix}-6 & -3\\ 4 & 2\end{bmatrix}$

Now, $A(\text{adj} A)=\begin{bmatrix}2 & 3\\ -4 & -6\end{bmatrix}\begin{bmatrix}-6 & -3\\ 4 & 2\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ ...(i)

and $(\text{adj} A)A=\begin{bmatrix}-6 & -3\\ 4 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\ -4 & -6\end{bmatrix}$ ...(ii)

Similarly, $|A|I=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ ...(iii) ($\because |A|=0$)

From equation (i), (ii) and (iii);

we have $A(\text{adj } A)=(\text{adj } A)\,A=|A|I$

43. Given, $A=\begin{bmatrix}1 & 0\\-4 & 2\end{bmatrix}$ and $B^{-1}=\begin{bmatrix}3 & 1\\5 & 2\end{bmatrix}$

Now, $|A|=\begin{vmatrix}1 & 0\\-4 & 2\end{vmatrix}=2$ and adj $A=\begin{bmatrix}2 & 4\\0 & 1\end{bmatrix}'=\begin{bmatrix}2 & 0\\4 & 1\end{bmatrix}$

$\therefore\quad A^{-1}=\frac{1}{|A|}(\text{adj }A)=\frac{1}{2}\begin{bmatrix}2 & 0\\4 & 1\end{bmatrix}$

Now, $(AB)^{-1}=B^{-1}A^{-1}$

$=\frac{1}{2}\begin{bmatrix}3 & 1\\5 & 2\end{bmatrix}\begin{bmatrix}2 & 0\\4 & 1\end{bmatrix}=\frac{1}{2}\begin{bmatrix}10 & 1\\18 & 2\end{bmatrix}=\begin{bmatrix}5 & 1/2\\9 & 1\end{bmatrix}$

44. We have, $A=\begin{bmatrix}2 & -3\\-4 & 7\end{bmatrix}$

$\therefore\quad |A|=\begin{vmatrix}2 & -3\\-4 & 7\end{vmatrix}=14-12=2\neq 0$

So, A is a non-singular matrix and therefore it is invertible.

$\therefore\quad \text{adj }A=\begin{bmatrix}7 & 4\\3 & 2\end{bmatrix}'=\begin{bmatrix}7 & 3\\4 & 2\end{bmatrix}$

Hence, $A^{-1}=\frac{1}{|A|}\text{adj }A=\frac{1}{2}\begin{bmatrix}7 & 3\\4 & 2\end{bmatrix}$

$\Rightarrow\ 2A^{-1}=\begin{bmatrix}7 & 3\\4 & 2\end{bmatrix}$ ...(i)

Now, $9I-A=9\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}-\begin{bmatrix}2 & -3\\-4 & 7\end{bmatrix}$

$=\begin{bmatrix}9 & 0\\0 & 9\end{bmatrix}-\begin{bmatrix}2 & -3\\-4 & 7\end{bmatrix}=\begin{bmatrix}7 & 3\\4 & 2\end{bmatrix}$ [From (ii)]

Hence, $2A^{-1}=9I-A$.

45. (i) Let the cost of each pen, each bag and each instrument box be x, y and z respectively.

According to question, we have

$5x+3y+z=160$ ....(i)

$2x+y+3z=190$ ...(ii)

$x+2y+4z=250$ ...(iii)

$\Rightarrow\ \begin{bmatrix}5 & 3 & 1\\2 & 1 & 3\\1 & 2 & 4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}160\\190\\250\end{bmatrix}$

i.e., $AX=B$

(ii) Now, $|A|=\begin{vmatrix}5 & 3 & 1\\2 & 1 & 3\\1 & 2 & 4\end{vmatrix}$

$=5(4-6)-3(8-3)+1(4-1)=-10-15+3=-22$

(iii) $\therefore\ A^{-1}=\frac{\text{adj }A}{|A|}=\frac{-1}{22}\begin{bmatrix}-2 & -10 & 8\\-5 & 19 & -13\\3 & -7 & -1\end{bmatrix}$

$=\begin{bmatrix}1/11 & 5/11 & -4/11\\5/22 & -19/22 & 13/22\\-3/22 & 7/22 & 1/22\end{bmatrix}$

OR

$A^2=\begin{bmatrix}5 & 3 & 1\\2 & 1 & 3\\1 & 2 & 4\end{bmatrix}\begin{bmatrix}5 & 3 & 1\\2 & 1 & 3\\1 & 2 & 4\end{bmatrix}=\begin{bmatrix}32 & 20 & 18\\15 & 13 & 17\\13 & 13 & 23\end{bmatrix}$

$P=A^2-5A$

$=\begin{bmatrix}32 & 20 & 18\\15 & 13 & 17\\13 & 13 & 23\end{bmatrix}-\begin{bmatrix}25 & 15 & 5\\10 & 5 & 15\\5 & 10 & 20\end{bmatrix}=\begin{bmatrix}7 & 5 & 13\\5 & 8 & 2\\8 & 3 & 3\end{bmatrix}$

46. Given $A^{-1}=\begin{bmatrix}3 & -1 & 1\\-15 & 6 & -5\\5 & -2 & 2\end{bmatrix}, B=\begin{bmatrix}1 & 2 & -2\\-1 & 3 & 0\\0 & -2 & 1\end{bmatrix}$

Here, $|B|=\begin{vmatrix}1 & 2 & -2\\-1 & 3 & 0\\0 & -2 & 1\end{vmatrix}=1(3-0)-2(-1-0)-2(2-0)$

$=3+2-4=1$

$\Rightarrow\ \text{adj}B=\begin{bmatrix}3 & 1 & 2\\2 & 1 & 2\\6 & 2 & 5\end{bmatrix}'=\begin{bmatrix}3 & 2 & 6\\1 & 1 & 2\\2 & 2 & 5\end{bmatrix}$

$\therefore\quad B^{-1}=\frac{1}{|B|}(\text{adj}B)=\begin{bmatrix}3 & 2 & 6\\1 & 1 & 2\\2 & 2 & 5\end{bmatrix}$

As we know that $(AB)^{-1}=B^{-1}A^{-1}$

$\therefore\quad B^{-1}A^{-1}=\begin{bmatrix}3 & 2 & 6\\1 & 1 & 2\\2 & 2 & 5\end{bmatrix}\begin{bmatrix}3 & -1 & 1\\-15 & 6 & -5\\5 & -2 & 2\end{bmatrix}=\begin{bmatrix}9 & -3 & 5\\-2 & 1 & 0\\1 & 0 & 2\end{bmatrix}$

47. $A=\begin{bmatrix}1 & -2 & 3\\0 & -1 & 4\\-2 & 2 & 1\end{bmatrix}\Rightarrow A'=\begin{bmatrix}1 & 0 & -2\\-2 & -1 & 2\\3 & 4 & 1\end{bmatrix}$

$|A'|=\begin{vmatrix}1 & 0 & -2\\-2 & -1 & 2\\3 & 4 & 1\end{vmatrix}=1(-1-8)-2(-8+3)$

$=-9+10=1\neq 0.$

So, $(A')^{-1}$ exists.

Let the cofactors of a_{ij}'s are A_{ij} in A'

Now, $A_{11}=-9, A_{12}=8, A_{13}=-5,$

$A_{21}=-8, A_{22}=7, A_{23}=-4,$

$A_{31}=-2, A_{32}=2, A_{33}=-1$

$\therefore\quad \text{adj}(A')=\begin{bmatrix}-9 & -8 & -2\\8 & 7 & 2\\-5 & -4 & -1\end{bmatrix}$

$\therefore\ (A')^{-1}=\frac{\text{adj}(A')}{|A'|}=\begin{bmatrix}-9 & -8 & -2\\8 & 7 & 2\\-5 & -4 & -1\end{bmatrix}$

48. Here, $A=\begin{bmatrix}-1 & -2 & -2\\2 & 1 & -2\\2 & -2 & 1\end{bmatrix}$

$\Rightarrow\quad |A|=\begin{vmatrix}-1 & -2 & -2\\2 & 1 & -2\\2 & -2 & 1\end{vmatrix}$

$= -1(1-4)-(-2)(2+4)-2(-4-2)$
$= 3+12+12 = 27$
Now, $A_{11} = -3, A_{12} = -6, A_{13} = -6,$
$A_{21} = 6, A_{22} = 3, A_{23} = -6,$
$A_{31} = 6, A_{32} = -6, A_{33} = 3$

$$\therefore \text{ adj } A = \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}' = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$\therefore \quad A\cdot(\text{adj } A) = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 27 \end{bmatrix} = 27 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = |A| I_3.$$

49. $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$$\Rightarrow \quad A^2 = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} \quad \text{...(i)}$$

$$\text{Now, } 4A - 3I = 4\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad 4A - 3I = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} \quad \text{...(ii)}$$

From (i) and (ii), we get
$A^2 = 4A - 3I$
Pre-multiplying by A^{-1} on both sides, we get
$A^{-1}(A^2) = 4A^{-1}A - 3A^{-1}I$
$\Rightarrow \quad A = 4I - 3A^{-1}$ $\quad [\because AA^{-1} = I]$
$\Rightarrow \quad 3A^{-1} = 4I - A$

$$\Rightarrow \quad A^{-1} = \frac{4}{3}I - \frac{1}{3}A$$

$$= \frac{4}{3}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$

50. Given $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

Now, $A_{11} = 7, A_{12} = -1, A_{13} = -1,$
$A_{21} = -3, A_{22} = 1, A_{23} = 0,$
$A_{31} = -3, A_{32} = 0, A_{33} = 1$

$$\therefore \text{ adj } A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\text{and } |A| = \begin{vmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{vmatrix}$$

$= 1(16-9) - 3(4-3) + 3(3-4) = 7 - 3 - 3 = 1 \neq 0$
So, A^{-1} exists and it is given by

$$A^{-1} = \frac{1}{|A|}\cdot \text{adj } A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}.$$

51. Here, $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$

$\Rightarrow \quad |A| = -11, |B| = 1$

$$\therefore \quad A^{-1} = \frac{1}{|A|}\text{adj } A = -\frac{1}{11}\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix} \text{ and } B^{-1} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$\therefore \quad$ R.H.S. $= B^{-1}A^{-1}$

$$= \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\left(-\frac{1}{11}\right)\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix} = \left(-\frac{1}{11}\right)\begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix} \quad \text{...(i)}$$

$$\text{Now, } A\cdot B = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}\cdot\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 5 & -14 \end{bmatrix}$$

$\Rightarrow \quad |AB| = 14 - 25 = -11$

$$\therefore \quad \text{L.H.S.} = (AB)^{-1} = \left(-\frac{1}{11}\right)\begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix} \quad \text{...(ii)}$$

From (i) and (ii), $(AB)^{-1} = B^{-1}A^{-1}$.

52. Given, $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now, $A_{11} = \cos\alpha, A_{12} = -\sin\alpha, A_{13} = 0,$
$A_{21} = \sin\alpha, A_{22} = \cos\alpha, A_{23} = 0,$
$A_{31} = 0, A_{32} = 0, A_{33} = 1$

$$\therefore \text{ adj}(A) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A.\text{adj}(A) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{...(i)}$$

$$\text{adj}(A)\cdot A = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{...(ii)}$$

$$|A| = \begin{vmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$= \cos\alpha(\cos\alpha - 0) + \sin\alpha(\sin\alpha - 0) + 0 = 1$ $\quad$ ...(iii)
From (i), (ii) and (iii), we get
$A(\text{adj } A) = (\text{adj } A)A = |A| I_3.$

53. Given, $A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}$

$|A| = 3(12-6) - 4(0+3) + 2(0-2)$
$= 18 - 12 - 4 = 2 \neq 0$

So, A^{-1} exists and $A^{-1} = \dfrac{1}{|A|}\text{ adj}(A)$

Now, $\text{adj}\,A=\begin{bmatrix}6 & -3 & -2\\ -28 & 16 & 10\\ -16 & 9 & 6\end{bmatrix}'=\begin{bmatrix}6 & -28 & -16\\ -3 & 16 & 9\\ -2 & 10 & 6\end{bmatrix}$

$\Rightarrow A^{-1}=\frac{1}{2}\begin{bmatrix}6 & -28 & -16\\ -3 & 16 & 9\\ -2 & 10 & 6\end{bmatrix}$ ...(i)

Given,

$3x+4y+2z=8$

$2y-3z=3$

$x-2y+6z=-2$

Given system of equations can be written as $AX=B$,

where $A=\begin{bmatrix}3 & 4 & 2\\ 0 & 2 & -3\\ 1 & -2 & 6\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $B=\begin{bmatrix}8\\ 3\\ -2\end{bmatrix}$

$X=A^{-1}B$

$=\frac{1}{2}\begin{bmatrix}6 & -28 & -16\\ -3 & 16 & 9\\ -2 & 10 & 6\end{bmatrix}\begin{bmatrix}8\\ 3\\ -2\end{bmatrix}=\frac{1}{2}\begin{bmatrix}48-84+32\\ -24+48-18\\ -16+30-12\end{bmatrix}$

$=\frac{1}{2}\begin{bmatrix}-4\\ 6\\ 2\end{bmatrix}=\begin{bmatrix}-2\\ 3\\ 1\end{bmatrix}$

$\therefore\quad x=-2, y=3, z=1$

54. Let the monthly income of Aryan be ₹ $3x$ and that of Babban be ₹ $4x$.

Also, let monthly expenditure of Aryan be ₹ $5y$ and that of Babban be ₹ $7y$.

According to question,

$3x-5y=15000$

$4x-7y=15000$

These equations can be written as $AX=B$

where, $A=\begin{bmatrix}3 & -5\\ 4 & -7\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}$ and $B=\begin{bmatrix}15000\\ 15000\end{bmatrix}$

$|A|=\begin{vmatrix}3 & -5\\ 4 & -7\end{vmatrix}=(-21+20)=-1\neq 0$

Thus, A^{-1} exists. So, system of equations has a unique solution and given by $X=A^{-1}B$

Now, $\text{adj}(A)=\begin{bmatrix}-7 & 5\\ -4 & 3\end{bmatrix}$

$\therefore\quad A^{-1}=\frac{\text{adj}(A)}{|A|}=\begin{bmatrix}7 & -5\\ 4 & -3\end{bmatrix}$

Now, $X=A^{-1}B=\begin{bmatrix}7 & -5\\ 4 & -3\end{bmatrix}\begin{bmatrix}15000\\ 15000\end{bmatrix}$

$\Rightarrow \begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}30000\\ 15000\end{bmatrix}\Rightarrow x=30000$ and $y=15000$

So, monthly income of Aryan $=3\times 30000$
$=$ ₹90000

Monthly income of Babban $=4\times 30000=$ ₹120000

From this question we are encouraged to save a part of money every month.

Key Points

- When $|A|\neq 0$, then A^{-1} exists, so the system of equations has a unique solution given by $X=A^{-1}B$.

55. Let ₹ x be invested in the first bond and ₹ y be invested in the second bond.

According to question,

$\frac{10x}{100}+\frac{12y}{100}=2800\Rightarrow 10x+12y=280000$ ...(i)

If the rate of interest had been interchanged, then the total interest earned is ₹ 100 less than the previous interest *i.e.*, ₹ 2700.

$\therefore\ \frac{12x}{100}+\frac{10y}{100}=2700\Rightarrow 12x+10y=270000$ ...(ii)

The system of equations (i) and (ii) can be represented as

$AX=B$, where $A=\begin{bmatrix}10 & 12\\ 12 & 10\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}280000\\ 270000\end{bmatrix}$

$|A|=\begin{vmatrix}10 & 12\\ 12 & 10\end{vmatrix}=100-144=-44\neq 0$

Thus A^{-1} exists. So, system of equations has a unique solution and given by $X=A^{-1}B$

$\text{adj}\,A=\begin{bmatrix}10 & -12\\ -12 & 10\end{bmatrix}$

Now, $X=A^{-1}B\Rightarrow X=\frac{\text{adj}\,A}{|A|}B$

$\Rightarrow \begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{(-44)}\begin{bmatrix}10 & -12\\ -12 & 10\end{bmatrix}\begin{bmatrix}280000\\ 270000\end{bmatrix}$

$\Rightarrow \begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{(-44)}\begin{bmatrix}-440000\\ -660000\end{bmatrix}=\begin{bmatrix}10000\\ 15000\end{bmatrix}$

$\Rightarrow\quad x=10000$ and $y=15000$

Therefore, ₹ 10,000 be invested in the first bond and ₹ 15,000 be invested in the second bond. Thus, the total amount invested by the trust = 10,000 + 15,000 = ₹ 25,000. The interest received will be given to Helpage India as donation reflects the helping and caring nature of the trust.

56. Let the monthly fees paid by poor and rich children be ₹ x and ₹ y respectively.

For batch I : $20x+5y=9000$ ...(i)

For batch II : $5x+25y=26000$ ...(ii)

The system of equations (i) and (ii) can be written as $AX=B$

where, $A=\begin{bmatrix}20 & 5\\ 5 & 25\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}$ and $B=\begin{bmatrix}9000\\ 26000\end{bmatrix}$

$|A|=\begin{vmatrix}20 & 5\\ 5 & 25\end{vmatrix}=500-25=475\neq 0$

Thus, A^{-1} exists. So, the given system has a unique solution and it is given by $X=A^{-1}B$.

$\text{adj}\,A=\begin{bmatrix}25 & -5\\ -5 & 20\end{bmatrix}$

$A^{-1}=\frac{\text{adj}\,A}{|A|}=\frac{1}{475}\begin{bmatrix}25 & -5\\ -5 & 20\end{bmatrix}$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{475}\begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}\begin{bmatrix} 9000 \\ 26000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{475}\begin{bmatrix} 95000 \\ 475000 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 200 \\ 1000 \end{bmatrix}$$

$\Rightarrow \quad x = 200, y = 1000$

Hence, the monthly fees paid by each poor child is ₹ 200 and the monthly fees paid by each rich child is ₹ 1000.

By offering discount to the poor children, the coaching institute offers an unbiased chance for the development and enhancement of the weaker section of our society.

57. (i) Given, value of prize for team spirit = ₹ x

Value of prize for truthfulness = ₹ y

Value of prize for tolerance = ₹ z

Linear equation for School A is $3x + y + 2z = 1100$

Linear equation for School B is $x + 2y + 3z = 1400$

Linear equation for Prize is $x + y + z = 600$

The corresponding matrix equation is $PX = Q$

where, $P = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $Q = \begin{bmatrix} 1100 \\ 1400 \\ 600 \end{bmatrix}$

(ii) Now $|P| = \begin{vmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix}$

$= 3\cdot(2-3) - 1\cdot(1-3) + 2\cdot(1-2)$

$= -3 + 2 - 2 = -3 \neq 0$

Thus, P^{-1} exists. So, system of equations has unique solution and it is given by $X = P^{-1}Q$

Now, cofactors of elements of P are

$A_{11} = -1, A_{12} = 2, A_{13} = -1,$
$A_{21} = 1, A_{22} = 1, A_{23} = -2,$
$A_{31} = -1, A_{32} = -7, A_{33} = 5$

$$\therefore \ \text{adj}\,P = \begin{bmatrix} -1 & 1 & -1 \\ 2 & 1 & -7 \\ -1 & -2 & 5 \end{bmatrix}$$

$$P^{-1} = \frac{1}{|P|}\cdot\text{adj}\,P = -\frac{1}{3}\begin{bmatrix} -1 & 1 & -1 \\ 2 & 1 & -7 \\ -1 & -2 & 5 \end{bmatrix}$$

Now, $X = P^{-1}Q$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} -1 & 1 & -1 \\ 2 & 1 & -7 \\ -1 & -2 & 5 \end{bmatrix}\begin{bmatrix} 1100 \\ 1400 \\ 600 \end{bmatrix}$$

$$= -\frac{1}{3}\begin{bmatrix} -300 \\ -600 \\ -900 \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix} \Rightarrow x = 100, y = 200, z = 300.$$

Thus, the above system of equations is solvable.

(iii) The value truthfulness should be rewarded the most because a student who is truthful will be also tolerant and will work with a team spirit in the school.

58. Given $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

$|A| = 1(8-6) + 1(0+9) + 2(0-6) = 2 + 9 - 12 = -1$

$\therefore \quad |A| \neq 0$, so A^{-1} exists

Now, Co-factors are

$A_{11} = 2, A_{12} = -9, A_{13} = -6, \quad A_{21} = 0, \quad A_{22} = -2$
$A_{23} = -1, A_{31} = -1, A_{32} = 3, \quad A_{33} = 2$

$$\text{Now, } adj(A) = \begin{bmatrix} 2 & -9 & -6 \\ 0 & -2 & -1 \\ -1 & 3 & 2 \end{bmatrix}^T = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

We know that $A^{-1} = \frac{1}{|A|}adj(A)$

$$= -1\begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

We know that $AX = B \Rightarrow X = A^{-1}B$ where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$

and $A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -2+0+3 \\ 9+2-9 \\ 6+1-6 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

Hence $x = 1, y = 2$ and $z = 1$

59. Given, $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{vmatrix} = 1(2-2) - 2(3+4) - 3(-3-4)$

$= -14 + 21 = 7 \neq 0$

$\therefore \quad A^{-1}$ exists

Now, $A_{11} = 0, A_{12} = -7, A_{13} = -7, A_{21} = 1, A_{22} = 7,$
$A_{23} = 5, A_{31} = 2, A_{32} = -7, A_{33} = -4$

$$\therefore \quad \text{adj}\,A = \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$$

$$\Rightarrow \quad A^{-1} = \frac{1}{|A|}\text{adj}\,A = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$$

The given system of equations is

$x + 2y - 3z = 6$
$3x + 2y - 2z = 3$
$2x - y + z = 2$

The system of equations can be written as $AX = B$

where $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$

$\because$ A^{-1} exists, so system of equations has a unique solution given by $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 7 \\ -35 \\ -35 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}$$

$\Rightarrow$ $x = 1, y = -5, z = -5$

60. We have, $x - y + 2z = 7$

$2x - y + 3z = 12$

$3x + 2y - z = 5$

The given system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 \\ 12 \\ 5 \end{bmatrix}$$

Here, $|A| = \begin{vmatrix} 1 & -1 & 2 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{vmatrix}$

$= 1(1 - 6) + 1(-2 - 9) + 2(4 + 3)$

$= -5 - 11 + 14 = -2 \neq 0$

$\therefore$ A^{-1} exists. So system of equations has a unique solution given by $X = A^{-1}B$

$$\therefore \text{adj } A = \begin{bmatrix} -5 & 11 & 7 \\ 3 & -7 & -5 \\ -1 & 1 & 1 \end{bmatrix}' = \begin{bmatrix} -5 & 3 & -1 \\ 11 & -7 & 1 \\ 7 & -5 & 1 \end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{-1}{2}\begin{bmatrix} -5 & 3 & -1 \\ 11 & -7 & 1 \\ 7 & -5 & 1 \end{bmatrix}$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} -5 & 3 & -1 \\ 11 & -7 & 1 \\ 7 & -5 & 1 \end{bmatrix}\begin{bmatrix} 7 \\ 12 \\ 5 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} -4 \\ -2 \\ -6 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$\Rightarrow$ $x = 2, y = 1, z = 3$

61. We have, $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$\Rightarrow$ $|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$

$= 2(-4 + 4) + 3(-6 + 4) + 5(3 - 2)$

$= -6 + 5 = -1 \neq 0$

$\therefore$ A^{-1} exists.

Now, $A_{11} = 0$, $A_{12} = 2$, $A_{13} = 1$, $A_{21} = -1$, $A_{22} = -9$, $A_{23} = -5$, $A_{31} = 2$, $A_{32} = 23$, $A_{33} = 13$

$$\therefore \text{adj } A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|}\text{adj } A = (-1)\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

The given system of equations is

$2x - 3y + 5z = 11, 3x + 2y - 4z = -5, x + y - 2z = -3$

The system of equations can be written as $AX = B$,

where $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

Since A^{-1} exists, therefore, system of equations has a unique solution given by

$$X = A^{-1}B = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\Rightarrow$ $x = 1, y = 2$ and $z = 3$.

62. Given, $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

$\Rightarrow$ $|A| = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = 0(2 - 3) - 1(1 - 9) + 2(1 - 6)$

$= 8 - 10 = -2$

As $|A| \neq 0$. $\therefore$ A^{-1} exists and A^{-1} is given by $\frac{1}{|A|}(\text{adj } A)$

$$\text{adj } A = \begin{bmatrix} -1 & 8 & -5 \\ 1 & -6 & 3 \\ -1 & 2 & -1 \end{bmatrix}' = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = -\frac{1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix} \quad \text{...(i)}$$

Given,

$y + 2z = 5$

$x + 2y + 3z = 10$

$3x + y + z = 9$

The given system of equations can be written as $AX = B$

Where, $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 10 \\ 9 \end{bmatrix}$

$X = A^{-1}B$

by using equation (i), we get

$$X = \frac{-1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ 10 \\ 9 \end{bmatrix} \Rightarrow \frac{-1}{2}\begin{bmatrix} -5+10-9 \\ 40-60+18 \\ -25+30-9 \end{bmatrix} = \frac{-1}{2}\begin{bmatrix} -4 \\ -2 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

$\therefore$ $x = 2, y = 1, z = 2$

63. We have $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$

Now, $|A| = 1(0 - 2) - 1(1 - 6) + 1(1)$

$= -2 + 5 + 1 = 4 \neq 0$

$\therefore$ A^{-1} exists.

Now, $A_{11} = -2, A_{12} = 5, A_{13} = 1, A_{21} = 0, A_{22} = -2, A_{23} = 2, A_{31} = 2, A_{32} = -1, A_{33} = -1$

$$\therefore \text{adj} A = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}\text{adj} A = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

Now the given equations are

$x + y + z = 6$

$x + 0y + 2z = 7$

$3x + y + z = 12$

The given system of equations can be written as $AX = B$ where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$\because$ A^{-1} exists, so system has a unique solution given by $X = A^{-1}B$.

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} -12+0+24 \\ 30-14-12 \\ 6+14-12 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 12 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$\Rightarrow$ $x = 3, y = 1, z = 2$

64. The given system of equations is

$x + 2y - 3z = -4$

$2x + 3y + 2z = 2$

$3x - 3y - 4z = 11$

The system of equations can be written as $AX = B$

where, $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$

Now, $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$

$$\Rightarrow |A| = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{vmatrix}$$

$= 1(-12 + 6) - 2(-8 - 6) - 3(-6 - 9)$

$= -6 + 28 + 45 = 67 \neq 0$

$\therefore$ A^{-1} exists.

Now, $A_{11} = -6, A_{12} = 14, A_{13} = -15, A_{21} = 17, A_{22} = 5, A_{23} = 9, A_{31} = 13, A_{32} = -8, A_{33} = -1$

$$\therefore \text{adj} A = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\cdot\text{adj} A = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

As A^{-1} exists, so system of equations has a unique solution given by $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$= \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$\Rightarrow x = 3, y = -2, z = 1.$

Concept Applied

- For a system of equations, $AX = B$, if $|A| \neq 0$, then the given system of equations is consistent and has a unique solution.

65. We have, $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

$$\text{or } \begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & -2 \\ 2 & -3 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 9 & 6 \\ 0 & 2 & 1 \\ 1 & -3 & -2 \end{bmatrix}$$

Now the given system of equations is

$x + 3z = 9$

$-x + 2y - 2z = 4$

$2x - 3y + 4z = -3$

The system of equations can be written as $AX = B$

where, $A = \begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & -2 \\ 2 & -3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$

Since, A^{-1} exists, so system of equations has a unique solution, given by

$X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 9 & 6 \\ 0 & 2 & 1 \\ 1 & -3 & -2 \end{bmatrix}\begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} -18+36-18 \\ 8-3 \\ 9-12+6 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$

$\Rightarrow$ $x = 0, y = 5, z = 3$

66. We have, $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$$

$$\Rightarrow \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = I$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}^{-1} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

The given system of equations is

$x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$ and it can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

Here, $|A| = 1(-6 + 2) + 1(3 + 4) + 1(1 + 4)$

$= -4 + 7 + 5 = 8 \neq 0$

So, the given system of equations has a unique solution given by $X = A^{-1}B$.

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix} \Rightarrow x = 3, y = -2, z = -1$$

67. Let one pen of variety 'A' costs ₹ x, one pen of variety 'B' costs ₹ y and one pen of variety 'C' costs ₹ z.

According to question,

$x + y + z = 21$ (For Meenu)

$4x + 3y + 2z = 60$ (For Jeevan)

$6x + 2y + 3z = 70$ (For Shikha)

The given system of equations can be written as $AX = B$

$$\text{where, } A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{vmatrix} = 1(9-4) - 1(12-12) + 1(8-18) = -5 \neq 0$$

$\therefore$ A^{-1} exists and system of equations has a unique solution given by $X = A^{-1}B$.

Now, $A_{11} = 5, A_{12} = 0, A_{13} = -10,$

$A_{21} = -1, A_{22} = -3, A_{23} = 4,$

$A_{31} = -1, A_{32} = 2, A_{33} = -1$

$$\therefore \text{ adj } A = \begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{(-5)}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{(-5)}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}\begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{(-5)}\begin{bmatrix} -25 \\ -40 \\ -40 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix} \Rightarrow x = 5, y = 8, z = 8$$

$\therefore$ Cost of 1 pen of variety 'A' = ₹ 5

Cost of 1 pen of variety 'B' = ₹ 8

Cost of 1 pen of variety 'C' = ₹ 8

68. According to question, we have,

$3x + 2y + z = 1000$ ...(i)

$4x + y + 3z = 1500$ ...(ii)

$x + y + z = 600$ ...(iii)

The given system of equations can be written as $AX = B$

$$\text{where, } A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1000 \\ 1500 \\ 600 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = -5 \neq 0$$

$\therefore$ A is invertible and system of equations has a unique solution given by $X = A^{-1}B$

Now, $A_{11} = -2, A_{12} = -1, A_{13} = 3,$

$A_{21} = -1, A_{22} = 2, A_{23} = -1,$

$A_{31} = 5, A_{32} = -5, A_{33} = -5$

$$\therefore \text{ adj } A = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{-1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} 1000 \\ 1500 \\ 600 \end{bmatrix} = \frac{-1}{5}\begin{bmatrix} -500 \\ -1000 \\ -1500 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix} \Rightarrow x = 100, y = 200, z = 300$$

Hence the money awarded for discipline, politeness and punctuality are ₹ 100, ₹ 200 and ₹ 300 respectively.

Apart from the above three values schools can award children for sincerity.

69. According to question, we have

$3x + 2y + z = 1600$ ...(i)

$4x + y + 3z = 2300$ ...(ii)

$x + y + z = 900$ ...(iii)

The given system of equations can be written as $AX = B$

$$\text{where } A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = -5 \neq 0$$

$\therefore$ A is invertible and system of equations has a unique solution given by $X = A^{-1}B$

Now, $A_{11} = -2, A_{12} = -1, A_{13} = 3,$
$A_{21} = -1, A_{22} = 2, A_{23} = -1,$
$A_{31} = 5, A_{32} = -5, A_{33} = -5$

$$\therefore \quad \text{adj } A = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$\therefore \; A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{-1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix} = \frac{-1}{5}\begin{bmatrix} -1000 \\ -1500 \\ -2000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 200 \\ 300 \\ 400 \end{bmatrix}$$

$\Rightarrow \quad x = 200, y = 300, z = 400$

Hence, the money awarded for sincerity, truthfulness and helpfulness are ₹ 200, ₹ 300 and ₹ 400 respectively.

Apart from the above three values schools can award children for discipline.

70. According to question, we have

$3x + 2y + z = 2200$ ...(i)

$4x + y + 3z = 3100$ ...(ii)

$x + y + z = 1200$ ...(iii)

The given system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = -5 \neq 0$$

$\therefore$ A is invertible and system of equations has a unique solution given by $X = A^{-1}B$

Now, $A_{11} = -2, A_{12} = -1, A_{13} = 3,$
$A_{21} = -1, A_{22} = 2, A_{23} = -1,$
$A_{31} = 5, A_{32} = -5, A_{33} = -5$

$$\therefore \quad \text{adj } A = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$\therefore \; A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{-1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix} = \frac{-1}{5}\begin{bmatrix} -1500 \\ -2000 \\ -2500 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 300 \\ 400 \\ 500 \end{bmatrix}$$

$\Rightarrow \quad x = 300, y = 400, z = 500$

Hence, the money awarded for tolerance, kindness and leadership are ₹ 300, ₹ 400 and ₹ 500 respectively.

Apart from the above three values schools can award children for sincerity.

71. Let ₹ x, ₹ y and ₹ z be deposited at the rates of interest 5%, 8% and $8\frac{1}{2}$% respectively.

According to question,

$x + y + z = 7000$

$x - y = 0$

$$x \cdot \frac{5}{100} + y \cdot \frac{8}{100} + z \cdot \frac{17}{2} \times \frac{1}{100} = 550$$

$\Rightarrow \quad 10x + 16y + 17z = 110000$

The system of equations can be written as $AX = B$

$$\text{where, } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 10 & 16 & 17 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 7000 \\ 0 \\ 110000 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 10 & 16 & 17 \end{vmatrix} = 1(-17) - (17) + 1(16 + 10) = -8 \neq 0$$

$\therefore \quad A^{-1}$ exists. So, system of equations has a unique solution and it is given by $X = A^{-1}B$

Now, $A_{11} = -17, A_{12} = -17, A_{13} = 26,$
$A_{21} = -1, A_{22} = 7, A_{23} = -6,$
$A_{31} = 1, A_{32} = 1, A_{33} = -2$

$$\therefore \quad \text{adj } A = \begin{bmatrix} -17 & -1 & 1 \\ -17 & 7 & 1 \\ 26 & -6 & -2 \end{bmatrix}$$

$$\text{and } A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{-8}\begin{bmatrix} -17 & -1 & 1 \\ -17 & 7 & 1 \\ 26 & -6 & -2 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{8}\begin{bmatrix} -17 & -1 & 1 \\ -17 & 7 & 1 \\ 26 & -6 & -2 \end{bmatrix} \cdot \begin{bmatrix} 7000 \\ 0 \\ 110000 \end{bmatrix}$$

$$\text{adj}(A) = \begin{bmatrix} 4 & 0 & 0 \\ -4 & 4 & 0 \\ 0 & -4 & 4 \end{bmatrix}$$

$\Rightarrow \quad x = 1125 = y, z = 4750$

Answer Tips

- First, make the linear equations from given word problem and then solve the equations.

72. According to question, we have

$x+y+z=1500$

$2x+3y+4z=4600$

$3x+2y+3z=4100$

The system of equations can be written as $AX=B$

where, $A=\begin{bmatrix}1&1&1\\2&3&4\\3&2&3\end{bmatrix}; X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}1500\\4600\\4100\end{bmatrix}$

$|A|=\begin{vmatrix}1&1&1\\2&3&4\\3&2&3\end{vmatrix}$

$=1(9-8)-1(6-12)+1(4-9)$

$=1+6-5=2\neq 0$

$\therefore$ A^{-1} exists and so, system of equations has a unique solution given by $X=A^{-1}B$

Now, $A_{11}=1, A_{12}=6, A_{13}=-5,$

$A_{21}=-1, A_{22}=0, A_{23}=1,$

$A_{31}=1, A_{32}=-2, A_{33}=1$

$\therefore$ $\text{adj}\,A=\begin{bmatrix}1&-1&1\\6&0&-2\\-5&1&1\end{bmatrix}$

$A^{-1}=\frac{1}{|A|}\cdot\text{adj}\,A=\frac{1}{2}\begin{bmatrix}1&-1&1\\6&0&-2\\-5&1&1\end{bmatrix}$

Now, $X=A^{-1}B$

$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{2}\begin{bmatrix}1&-1&1\\6&0&-2\\-5&1&1\end{bmatrix}\begin{bmatrix}1500\\4600\\4100\end{bmatrix}$

$=\frac{1}{2}\begin{bmatrix}1000\\800\\1200\end{bmatrix}=\begin{bmatrix}500\\400\\600\end{bmatrix}\Rightarrow x=500; y=400; z=600.$

Apart from sincerity, truthfulness and hard work, the schools can include an award for regularity.

CBSE Sample Questions

1. (d) : We have, $\begin{vmatrix}2&4\\5&1\end{vmatrix}=\begin{vmatrix}2x&4\\6&x\end{vmatrix}$

$\Rightarrow$ $2-20=2x^2-24 \Rightarrow 2x^2=6 \Rightarrow x^2=3 \Rightarrow x=\pm\sqrt{3}$ (1)

2. (a) : $|AA'|=|A|\,|A'|=|A'|^2=(-3)^2=9$ (1)

3. (c): $\because$ A is singular matrix.

$\therefore$ $|A|=0$

$\Rightarrow \begin{vmatrix}k&8\\4&2k\end{vmatrix}=0 \Rightarrow 2k^2-32=0 \Rightarrow k^2=16 \Rightarrow k=\pm 4$ (1)

4. (c): We have, $A^2=2A$

$\Rightarrow$ $|A^2|=|2A|$

$\Rightarrow$ $|A|^2=2^3|A|$ [As $|kA|=k^n|A|$ for a matrix of order n]

$\Rightarrow$ $|A|\,[|A|-8]=0$

$\Rightarrow$ either $|A|=0$ or $|A|=8$

But A is non-singular matrix $\Rightarrow$ $|A|\neq 0$

$\therefore$ $|2A|=2^3\cdot|A|=64$ (1)

5. We have, $A^2=2A$

$\Rightarrow$ $|AA|=|2A|$

$\Rightarrow$ $|A||A|=8|A|$ (1/2)

($\because$ $|AB|=|A||B|$ and $|kA|=k^n|A|$, where n is order of square matrix A)

$\Rightarrow$ $|A|\,(|A|-8)=0$ (1)

$\Rightarrow$ $|A|=0$ or 8 (1/2)

6. (b): We have, $|A|=-7$

$\therefore \sum_{i=1}^{3} a_{i2}A_{i2}=a_{12}A_{12}+a_{22}A_{22}+a_{32}A_{32}=|A|=-7$ (1)

7. We know that, if elements of a row are multiplied with cofactors of any other row, then their sum is 0.

$\therefore$ $a_{11}A_{21}+a_{12}A_{22}+a_{13}A_{23}=0.$ (1)

8. (b) : Given, $|A|=5$, order of matrix, $n=3$.

$|\text{adj}\,A|=|A|^{n-1} \Rightarrow |\text{adj}\,A|=25$ (1)

9. (d): We know that, $|\text{adj}\,A|=|A|^{n-1}$, where n is the order of A.

Here, $|\text{adj}\,A|=|A|^2=(-4)^2=16$ (1)

10. (c) : We know that, $(\text{adj}\,A)'$ = cofactor matrix of A

Here, cofactor matrix of $A=\begin{bmatrix}7&11\\-5&2\end{bmatrix}=(\text{adj}\,A)'$ (1)

11. (b): We have, $|A|=6+1=7$

Also, $\text{adj}\,A=\begin{bmatrix}2&-1\\1&3\end{bmatrix}$

Now, $A^{-1}=\frac{1}{|A|}\text{adj}(A)=\frac{1}{7}\begin{bmatrix}2&-1\\1&3\end{bmatrix}$

$\therefore$ $14A^{-1}=\begin{bmatrix}4&-2\\2&6\end{bmatrix}$ (1)

12. We know, $|\text{adj}A|=|A|^{n-1}$, where $n\times n$ is the order of non-singular matrix A.

$\therefore$ $|\text{adj}\,A|=(-4)^{3-1}=16$ (1)

13. Given, $A=\begin{bmatrix}2&-3&5\\3&2&-4\\1&1&-2\end{bmatrix}$

$\therefore$ $|A|=2(0)+3(-2)+5(1)=-1\neq 0$ (1/2)

As, $|A|\neq 0$, so A^{-1} exists and given by

$A^{-1}=\frac{\text{adj}\,A}{|A|}$

Now, $\text{adj}A=\begin{bmatrix}0&-1&2\\2&-9&23\\1&-5&13\end{bmatrix}, A^{-1}=\frac{1}{(-1)}\begin{bmatrix}0&-1&2\\2&-9&23\\1&-5&13\end{bmatrix}$ (1½)

The given system of equations are :

$2x-3y+5z=11$

$3x+2y-4z=-5$

$x+y-2z=-3$

The given system of equations can be written as $AX=B$

where, $A=\begin{bmatrix}2&-3&5\\3&2&-4\\1&1&-2\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}11\\-5\\-3\end{bmatrix}$

$$\Rightarrow \quad X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{(-1)}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} \quad (1½)$$

$$= \frac{1}{(-1)}\begin{bmatrix} 0+5-6 \\ 22+45-69 \\ 11+25-39 \end{bmatrix} \quad (1/2)$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{(-1)}\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix} \Rightarrow x = 1, y = 2 \text{ and } z = 3. \quad (1)$$

14. We have, $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$

$\therefore \quad |A| = 1(-1-2) - 2(-2-0) + 0 = -3 + 4 = 1 \neq 0$

$\therefore \quad A^{-1}$ exists. (1)

Now, $\text{adj } A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

$$\therefore \quad A^{-1} = \frac{1}{|A|}(\text{adj } A) = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \quad (1½)$$

We have, $x - 2y = 10$

$2x - y - z = 8$

$-2y + z = 7$

The given equations can be written as

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}, \quad (1)$$

which is of the form $AX = B$

where, $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}$

$\Rightarrow \quad X = A^{-1}B$ (1)

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$\Rightarrow \quad x = 0, y = -5, z = -3$ (1½)

15. We have,

$$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} \quad (1½)$$

$\Rightarrow \quad AB = 6I \Rightarrow A^{-1} = \frac{1}{6}(B)$ (1)

The given equations can be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

which is of the form $AX = C$

$\Rightarrow \quad X = A^{-1}C$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} \quad (1)$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$\Rightarrow x = 2, y = -1, z = 4$ (1½)